Arguments for the presence of a distant large undiscovered Solar system planet

J. B. Murray[*]

Department of Earth Sciences, The Open University, Milton Keynes MK7 6AA

ABSTRACT

Aphelion distances of long-period comets show a slight excess around 30 000 to 50 000 au from the Sun. Positions of cometary aphelia within these distance limits are aligned along a great circle inclined to both the ecliptic and the Galactic plane. This paper examines one of the possible explanations for this non-random clustering: that it is due to orbital perturbations by an undiscovered object orbiting within the above-mentioned distances. A model consistent with the observations gives a retrograde orbit (inclination $120°$) for the object with a longitude of the ascending node at $77° \pm 13°$, a period of 5.8×10^6 yr and a radius of 32 000 au. The same model gives a present position for the undiscovered object of RA $20^h 35^m$, Dec. $+5°$, with an error ellipse semimajor axis of $14°$ and a semiminor axis of $7°$. The magnitude is likely to be fainter than 23. Such a distant object would almost certainly not remain bound for the age of the Solar system, and recent capture into the present orbit, although also of low probability, remains the least unlikely origin for this hypothetical planet.

Key words: comets: general – planets and satellites: general.

1 BACKGROUND

The continuing announcement of several new Kuiper belt objects orbiting at similar distances to Pluto and beyond is presently increasing the known population of the outer Solar system (Jewitt & Luu 1995). These objects orbit at solar distances between 35 and 50 au. In the present study, evidence for a large planet orbiting three orders of magnitude further out is considered.

2 APHELION CLUSTERING OF LONG-PERIOD COMETS

The catalogue of cometary orbits (Marsden & Williams 1994) lists reciprocal semimajor axes of the original orbit of the incoming comet before perturbation by the planetary system for 298 long-period comets that have been determined with sufficient accuracy. A histogram of the aphelion distances of these comets shows an excess centred around 40 000 au (Fig. 1a), which in the past has been taken to be evidence of the Oort Cloud (Kresak 1982). The accuracies of the catalogued orbits are divided into four classes, some of which may have errors in aphelion distance greater than 30 000 au. This would place their aphelia well away from the peak at 40 000 au, so to check whether the clustering of cometary aphelia is an artefact of the errors, a histogram of the most

[*]E-mail: j.b.murray@open.ac.uk

accurate orbits only is plotted (Fig. 1b). These have aphelion distance uncertainties of the order of 3000–6000 au, and the majority of them cannot be outside the range 30 000–50 000 au. These more accurate orbits, details of which are given in Table 1, show an even more well-defined peak at 40 000 au, justifying the conclusion of previous workers that the clustering is real.

Fig. 1(b) resembles the clustering of periodic comet aphelia outside and just inside the orbit of the planet Jupiter (Fig. 1c). Although other explanations are possible, in the present paper the hypothesis that the 40 000-au clustering is due to a distant unknown planet is examined. Aphelion positions have therefore been plotted for all class 1a comets with distances between 30 000 and 50 000 au. The resulting distribution of positions (Fig. 2) is not random. The aphelia are all within $40°$ of the ecliptic, but, more importantly, those between $0°$ and $180°$ longitude show a reasonable approximation to a sine curve, as would be expected if their orbits had been captured into their present configuration by the presence of an unknown distant object orbiting the Sun at an inclination to the ecliptic.

Fitting a sine curve to the aphelia between $0°$ and $180°$ gives an inclination of $30° \pm 7°$ for the orbit of the hypothetical object, and a longitude of the ascending node at $257° \pm 13°$ for a prograde orbit or $77° \pm 13°$ for a retrograde orbit. The orbit is inclined at $35°$ to the Galactic plane.

The perturbations suffered by cometary orbits in the inner Solar system make it likely that many of these comets have approached
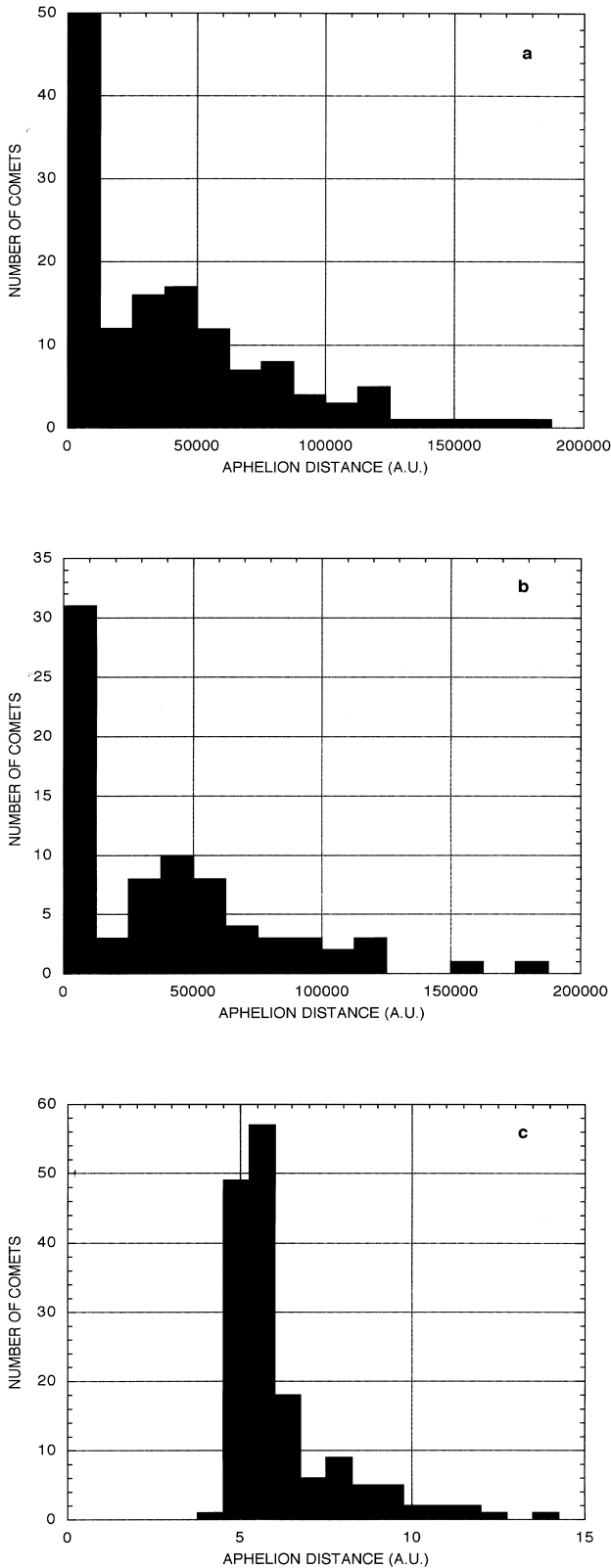
Figure 1. (a) Histogram of distances (in au) of long-period cometary aphelia listed in Marsden & Williams (1994). Note the higher numbers between 30 000 and 50 000 au from the Sun. (b) As (a), but using only the highest precision orbits. Note that the 30 000- to 50 000-au excess is even more pronounced. (c) As (a), but for periodic comets. Note the higher numbers between 4.5 and 6 au from the Sun. Jupiter orbits at a mean distance of 5.2 au from the Sun.

the Sun for the first time after perturbation by the unknown object. If this is the case, then examination of the aphelion longitudes and periods of the comets in Table 1 might give a clue to the present position of the object. The longitude L_h of the hypothetical planet may be taken, to a first approximation, as $L_A \approx L_h$ at time $t = -P/2$ for first-return comets, or $t = -[(n-1)P + (P/2)]$ for nth-return comets. (L_A is the longitude of comet aphelion.)

Although the scatter would be expected to be huge, as capture of objects with a variety of original orbital distances, inclinations and eccentricities is possible, a 'sawtooth' appearance in a plot of L_A versus P, owing to a progressive trend in the orbital periods of the comets along short sections of the supposed orbit, might indicate the direction of motion of the hypothetical object and reveal an extrapolated position. Fig. 3(a) is a plot of the original cometary periods against aphelion longitude for comets in Table 1. There is a suggestion of a sawtooth appearance, the data dividing into two groups (0° to 130° and 180° to 270°) of comets, both showing a possible correlation between increasing period and increasing longitude, although there is considerable scatter and the correlation coefficient of both groups is poor (0.60 for group 1 and 0.48 for group 2). This points to a retrograde motion for the unknown object; all but two of the comets in Table 1 also have a retrograde motion.

In this model, group 1 comets are on their first return to the Sun after capture, and group 2 are on their second return; Fig. 3(b) is plotted on this assumption. Published values (Marsden & Williams 1994) of the reciprocal semimajor axis before entering and after leaving the planetary region show that most orbits suffer considerable changes arising from planetary perturbations, hence the poorer correlation of group 2 comets. Plotting L_h against t for the comets in Table 1, a least-squares fit is obtained,

$$t = a + bL_h, \qquad (1)$$

from which, if longitude is given in degrees, the period P_h of the hypothetical planet is equal to

$$P_h = 360b.$$

This gives an orbital period for the hypothetical object of 5.8×10^6 yr. Its angular velocity v/D_h is thus known. Assuming a circular orbit, and ignoring the mass of the comets and the unknown planet, the distance D_h from the Sun is derived with sufficient accuracy from

$$D_h \approx [GM_\odot/(v/D_h)^2]^{1/3},$$

where G is the constant of gravitation, $M_\odot$ is the mass of the Sun, and v is the orbital velocity. From the above equation, the derived distance D_h for the object is 32 300 au. From equation (1), the present longitude of the object in degrees is given by

$$L_h = -360(a/P_h).$$

This places the object at ecliptic longitude 314° ± 14°, and the inclination derived earlier places it at ecliptic latitude +28° ± 7°, or RA 20^h 35^m, Dec. +5° at the present time.

Assuming that the undiscovered object exists, there still remains great uncertainty as to its present position. The distance of the object from the Sun is based on orbital motion which in turn is based on selection of cometary first and second solar approaches after capture from Fig. 3(a). An alternative approach would be to gauge the distance from the distribution of cometary aphelia in Fig. 1(b), which might place the orbit closer to 40 000 au from the Sun, in which case the present position would be more than 10° back in its orbit.

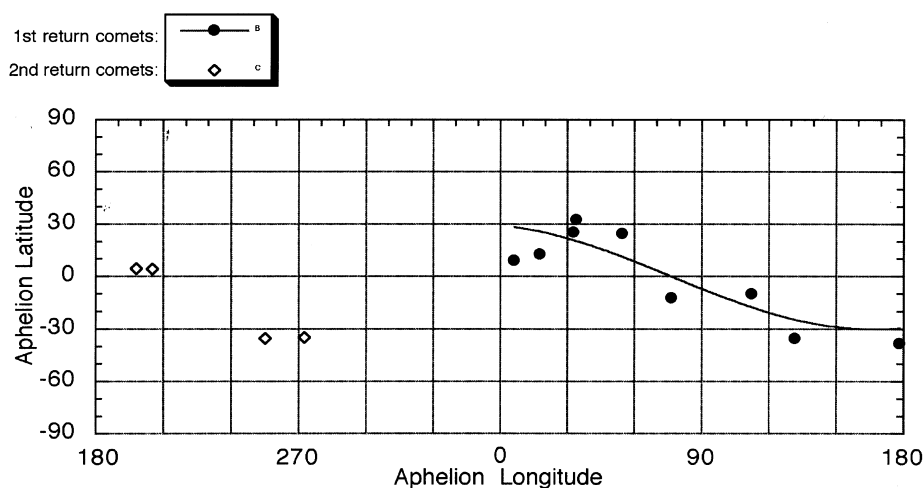The cometary aphelia from which the orbit of the object has

Figure 2. Plot of aphelion positions for all high-precision cometary orbits with aphelia between 30 000 and 50 000 au from the Sun listed in Marsden & Williams (1994). Note the apparent non-random distribution of those between 0° and 180° (filled circles). The curve is the best-fitting sine function to these points, which are assumed to be comets on their first return to the inner Solar system after capture by the hypothetical distant planet.

Table 1. Comets with aphelia between 30 000 and 50 000 au (from Marsden & Williams 1994). Columns from left to right: comet identification; q = perihelion distance; ω = argument of perihelion; Ω = longitude of ascending node; i = inclination; L_A = longitude of aphelion; B_A = latitude of aphelion; Q = aphelion distance. All angles are in degrees; distances are in astronomical units.

Comet	q	ω	Ω	i	L_A	B_A	Q
1925 I	1.109	36.2	319.1	100.0	131.9	−35.5	50 000
1946 VI	1.136	320.4	238.3	57.0	34.1	+32.3	45 000
1990 VI	1.569	137.8	280.0	59.4	255.2	−35.3	41 000
1889 I	1.815	340.5	359.0	166.4	198.0	+4.5	42 000
1993a	1.937	130.7	144.7	124.9	178.4	−38.5	33 000
1932 VI	2.314	329.7	216.1	125.0	54.6	+24.4	44 000
1972 VIII	2.511	167.9	358.9	138.6	6.3	+9.0	41 000
1983 XII	3.318	186.2	209.6	134.7	205.2	+4.4	44 000
1987 V	3.625	329.1	194.5	124.1	33.0	+25.2	34 000
1955 VI	3.87	144.7	265.3	100.4	272.6	−34.7	48 000
1979 VI	4.687	10.1	293.1	92.2	112.7	−10.1	48 000
1986 XIV	5.458	17.0	268.3	132.5	76.7	−12.5	43 000
1975 II	6.881	193.4	22.8	112.0	17.7	+12.4	34 000

been determined show more scatter than those of the short-period comets associated with the planet Jupiter. One comet in Fig. 1 shows an aphelion 19° from the orbit of the hypothetical object; there are few comets in Jupiter's family with aphelia more than 12° from Jupiter's orbit. However, comets associated with Jupiter are likely to have undergone several refinements and changes of orbit, whereas the cometary orbits considered here are first or second solar approaches only. The retrograde motion of all but two of these comets is shared by the hypothetical object.

3 DISCUSSION

The hypothetical planet is within the distance range where large numbers of small planets are predicted (Stern 1991), but the presence of a large object orbiting so far from the Sun would be surprising, as it would be very weakly bound and is extremely unlikely to have been an original member of the Solar system. On the other hand, recent capture of an object into a bound orbit at

this distance, although possible, is also extremely unlikely. The number of cometary orbits of sufficient accuracy for the present analysis is small, only 13, but the probability of the cluster in Fig. 2 occurring by chance is less than 0.0006 (see the appendix).

It is possible that some of these comets were not perturbed by the unknown planet and lie within the same area by chance. Some comets could be rejected from the analysis because they are furthest from the supposed orbit, and this would produce higher correlation coefficients and apparently less uncertain orbital elements and present position. However, such rejection would necessarily be subjective, as there is no way of distinguishing such comets.

The scenario that the alignment is due to a recent single approach of a perturber on a hyperbolic orbit, such as a passing star, is not possible, because the cluster in Fig. 2 extends to nearly 270°. If we reject comet 1993a and all second-return comets, then the arc is only about 130° and such a hypothesis might be tenable, but the same argument as in the previous paragraph applies: there is no objective reason for rejecting these comets. In the past, the possibility of a distant companion star to the Sun was discussed (Davis, Hut & Muller 1984; Whitmire & Jackson 1984), and one of the arguments in favour included the anisotropy in the positions of long-period cometary orbits (Delsemme 1986). However, because of its brightness, a much more distant and eccentric orbit than that indicated in Fig. 3 was implied. Furthermore, such a companion star is now considered extremely unlikely for a variety of other reasons (Vandervoort & Sather 1993). We are therefore left with an object smaller than a star, and at a distance that only comets are known to reach.

The mass of the hypothetical object would presumably be large compared with those of the known planets, in order to produce a detectable family of comets. However, the mass cannot be too large or it would be subject to energy-releasing nuclear reactions (Saumon et al. 1995) that would make the object too bright to have remained undiscovered. Assuming a diameter 10 times that of Jupiter, and a similar albedo, gives the object a visual magnitude fainter than 23.

An object as faint as this would be unlikely to have been picked up by any of the past searches for distant Solar system objects (Tombaugh 1961; Kowal 1989), because of its faintness and small
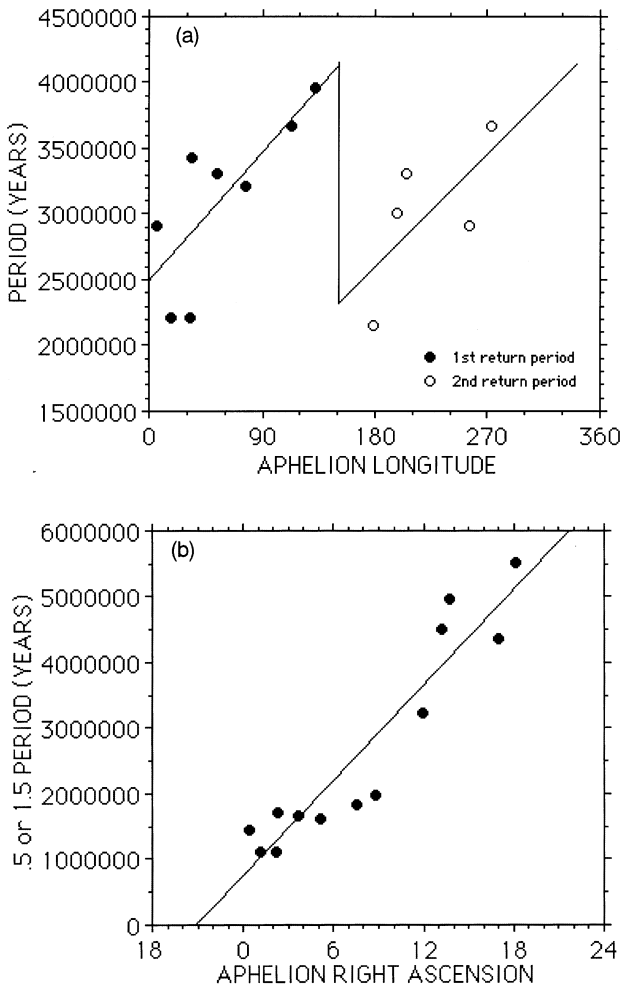
Figure 3. (a) Plot of period against aphelion longitude for the comets listed in Table 1 and shown in Fig. 2. Separate least-squares fits are shown for presumed first- and second-return comets. (b) Presumed time since capture for the comets listed in Table 1, plotted against RA. The time since capture is assumed to be 0.5 of the period for first-return comets, and 1.5 times the period for second-return comets. In this model, the point where the least-squares line crosses the *x*-axis indicates the present RA of the hypothetical object.

apparent motion. The orbital motion is less than 1 arcsec in 4 yr; the annual parallax is greater at 13 arcsec, giving an opposition motion of 1 arcsec in 9 d. Recent Earth-based searches for objects in the Kuiper belt have limiting magnitudes not faint enough (Levison & Duncan 1990) or on the limits of detectability (Jewitt & Luu 1995). Present *Hubble Space Telescope* searches can achieve 28th magnitude, but assume typical motions of 1 to 6 arcsec per hour; furthermore, they are deliberately directed at quadrature to eliminate the Earth's parallactic motion, and provide a direct measure of orbital motion (Cochran et al. 1995), so 1 arcsec in 4 yr would be several orders of magnitude too small to be detected.

However, a Jupiter-sized planet would emit more energy than it receives, so it is likely that the object would be significantly brighter in infrared, more so if it is larger than Jupiter; but the model position is also around 15° from the Galactic equator, where the increased background will hamper detection.

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APPENDIX

The bunching of the observed cometary aphelia into the band shown in Fig. 2 is apparently non-random. To work out the probability of this alignment happening by chance, all comets in Table 1 were used in the analysis. These fall within 30° of a great circle, i.e. they occupy 50 per cent of the celestial sphere. The chances of random points falling within half an available area are therefore considered.

The problem is slightly complicated by the fact that the band is not pre-determined, but is defined by the comets themselves. If the comets are numbered $1, 2, 3, \ldots, n$, then the position of comet 1 is unimportant, as it simply fixes an axis. The surface of the sphere may then be divided into two regions: caps and elsewhere. The caps are regions where points on the surface are at an angle of less than 30° relative to the axis. For points $2, 3, \ldots, n-1$, once a single point has fallen outside the cap, the position of a band around the great circle is then defined, so for all subsequent points the probability of them lying in or outside the band around the great circle is considered. $P(\text{point in fixed caps}) = \sqrt{3}/2$; $P(\text{point in fixed band}) = \frac{1}{2}$. So, for n ($n > 1$) comets,

$$P(\text{all } n \text{ comets lie in band}) = [1-(\sqrt{3}/2)]^{(n-2)} + \sqrt{3}[(1/2)^{(n-1)}]$$
$$\times \{[1-(2-\sqrt{3})]^{(n-2)}/[1-(2-\sqrt{3})]\}.$$

Therefore, for $n = 13$, $P(\text{all } n \text{ comets lie in band}) = 0.00058$. This is an upper limit for the probability of the clustering observed in Fig. 2, since the cluster extends through 270° only, and furthermore an irregular area could be drawn around the comets that takes up a great deal less than 50 per cent of the area of the sky.